

Low-scale SUSY breaking by modular fields and Higgs mass bounds

Satyanarayan Mukhopadhyay^{a *}, Biswarup Mukhopadhyaya^{a †}, Soumitra SenGupta^{b ‡}

^a*Regional Centre for Accelerator-based Particle Physics,
Harish-Chandra Research Institute, Chhatnag Road,
Jhusi, Allahabad - 211 019, India.*

^b*Department of Theoretical Physics,
Indian Association for the Cultivation of Science,
Kolkata-700 032, India.*

We consider a scenario where supersymmetry (SUSY) is broken at a relatively low scale by modular fields of extra compact spacelike dimensions. The effect of both soft and hard SUSY breaking terms on the mass of the lightest neutral Higgs boson are investigated. An important conclusion is that the lightest neutral Higgs can be considerably more massive than what is expected in the MSSM, if the overseeing theory breaks SUSY at a scale not too far above a TeV. An explicit model that implements this has been shown for illustration.

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I. INTRODUCTION

Supersymmetry (SUSY), as a cure to the quadratic divergence problem, requires embedding in a bigger canvas, in order to account for the many free parameters that appear in the soft SUSY-breaking Lagrangian. Also, such embedding enables one to generate phenomenologically consistent new particle spectra, bypassing the supertrace theorem. Some common schemes that are frequently considered include high scale SUSY breaking through supergravity (SUGRA), or SUSY breaking via gauge mediation (GMSB) at a somewhat lower scale, with the standard model (SM) gauge interactions yielding SUSY-breaking masses via loop-induced interaction with a messenger sector.

In order to be phenomenologically relevant, the new particles in the minimal SUSY standard model (MSSM) should have masses around the TeV scale. In spite of the exalted ‘top-down’ approach, it is somewhat disquieting that such a striking feature of nature as boson-fermion symmetry (as well as its controlled breaking) should be determined entirely by scales several orders of magnitude higher. Historically, as we have been able to access progressively higher energy scales, we have seen new laws of physics gradually unravelled. Furthermore, the physics of a particular scale is found to be mostly influenced by that in the scales immediately above it. The question is: when it comes to the outstanding questions of SUSY phenomenology at the TeV scale, wouldn’t it be fair to expect the decisive clues to lie around 10 TeV or so?

Let us look, for the sake of illustration, at two issues related to MSSM, following such a bottom-up philosophy. First, the SUSY-conserving Higgsino mass parameter μ occurring in the superpotential should naturally be very large if the MSSM is embedded in a theory valid upto a very large (Planck or Grand Unification) scale. The fact that phenomenology demands a μ within a TeV leads to a new naturalness problem. This ‘ μ -problem’ however, is not so acute anymore if MSSM emerges with SUSY breaking terms given by some effective theory, valid upto about 10 TeV. Secondly, according to accepted dicta, the non-observation of the lightest neutral Higgs upto about 140 GeV can almost rule out MSSM, for radiative corrections within the model cannot hike it beyond such a value. However, one may enquire whether some additional physics around 10 TeV can boost the lightest neutral Higgs mass to still higher values. It has indeed been claimed in an effective theory framework that higher-order terms in the MSSM fields, suppressed by a scale somewhat above a TeV, provide some upward revisions to the Higgs mass [1–4]. There have also been several studies in recent times, suggesting modifications to Higgs mass upper bounds when the standard model gauge group is extended by, for example, additional U(1) factors [5].

It should be noted that the potential threat from flavour changing neutral currents (FCNC) in SUGRA-type scenarios can be avoided if flavour diagonal soft SUSY breaking masses are generated at a relatively low scale. This is because the mass parameters then do not ‘run’ long enough to cause sufficient misalignment between the fermion and sfermion mass matrices.

The other point to remember is that one can in principle also allow hard SUSY-breaking terms in the Lagrangian [6]. Such terms may acquire particular significance when one has additional physics intervening at relatively low scale(s).

* E-mail: satya@hri.res.in

† E-mail: biswarup@hri.res.in

‡ E-mail: tpssg@iacs.res.in

The role of such terms in the context of neutrino masses [7] and also in other contexts of SUSY phenomenology [8] have been investigated in the literature. It has been shown that, with specific F-term SUSY breaking assumptions, the hard terms can cause radiative corrections to the Higgs mass which are at best of the magnitude of the soft SUSY breaking masses themselves [9]. In addition, as the scale of SUSY breaking is now low, these terms can give rise to substantial tree-level corrections to the MSSM Higgs quartic couplings [3, 9]. In a completely model-independent setting, however, the radiative corrections arising due to the presence of hard terms can be unacceptably large. On the other hand, if MSSM is embedded in an effective theory valid upto about 10 TeV, then hard SUSY breaking terms that are non-vanishing below such a scale are always safe from the viewpoint of the lightest neutral Higgs mass, irrespective of whether they are arising from F-term vacuum expectation values (vev) or not. They can, at the same time, be instrumental in contributing substantially and positively to the Higgs mass. If that happens, then the non-observation of the lightest neutral Higgs within the stipulated mass limit is possible, though one may see other signals of new physics at the Large Hadron Collider. We discuss such a possibility in this paper, in the context of a scenario based on extra compact spacelike dimensions in a string inspired higher dimensional supergravity model. As will be seen below, the essence of our proposal lies in treating the vev's of the modular fields for the extra dimensions as the cut-off of the low energy theory below TeV scale and therefore as a suppressant in the terms in the Kahler potential and the superpotential leading to soft SUSY breaking.

In any theory of this type, the pertinent question is: what provides the scale of 10 TeV in a sector beyond the MSSM spectrum? In the extra dimensional context, one usually thinks of the ‘bulk’ Planck scale which can be low. We consider an alternative possibility here. Whenever the additional spacelike dimensions are compactified, one ends up with modular fields whose vev's are related to the stable radii of these dimensions. There can be a number of modular fields, including the radii of the different compact dimensions and the angles among them. We suggest the possibility of the vev's of scalar components of the modular superfields as the additional scale. If these vev's are about 10 TeV, then the SUSY theory is effective below this scale, above which the modular fields develop as dynamical degrees of freedom. Below the energy scale corresponding to the vev of the modular fields, on the other hand, one can think of an effective description, with all higher-dimensional operators suppressed by the aforementioned scale. The dominant contributions to soft SUSY breaking terms in the observable sector may arise from such terms which take the place of terms suppressed by the Planck scale in common supergravity scenarios.

Here we suggest a KKLT-type [10] scenario where SUSY is broken by introducing a lifting term in the potential in terms of the modulus field T which lifts an N=1 SUSY anti-de Sitter vacuum to a Minkowski/de-Sitter vacuum with broken SUSY. The corresponding scalar potential has a well-defined minimum and the vev of the T field can be tuned at a scale near 10 TeV. The relevant F-term component is at an intermediate scale of around $\sim (3 \text{ TeV})^2$ and one can easily obtain the required low-energy SUSY spectrum with the soft masses $\sim 1 \text{ TeV}$. As has been already mentioned, if SUSY is broken at a relatively low scale, the effect of certain hard SUSY-breaking operators are no longer negligible, although they do not bring back any harmful quadratic divergences and can give finite and bounded contributions to observables like the Higgs mass. In order to see the implications of such a low-scale SUSY-breaking scenario, we shall study how the upper bounds on the lightest Higgs boson mass might be modified, thereby reducing the tension between this bound in MSSM and the limit coming from LEP.

The framework proposed by us is outlined more precisely in section II. In section III, a form of the potential that can lead to the appropriate vev's of the scalar and auxiliary components of the modular fields is suggested. The implications on the lightest neutral Higgs mass, including the effects of the hard terms, are shown in section IV, along with some numerical estimates. We summarise and conclude in section V.

II. THE GENERAL SCENARIO

In a generic SUSY breaking mechanism two different scales are involved. One is the SUSY breaking scale \sqrt{F} which corresponds to the vev's of the relevant auxiliary fields in the SUSY breaking sector. The other one is the M , associated with the interactions that transmit the breaking to the observable sector. In an extra dimensional model M is determined by the compactification scale which in turn is related to the inverse of the radius of the extra dimension. M thus sets the scale of the effective theory and as we are considering an effective theory below M , this transmission takes place through higher-dimensional operators suppressed by powers of M . For example, these operators give rise to the scalar soft masses of the form

$$m_{soft} \sim \frac{F}{M}. \quad (2.1)$$

In addition to the soft terms, generically hard SUSY breaking terms can also be present in the effective theory. For example, one can have a scalar quartic coupling of the form

$$\lambda_{hard} \sim \frac{F^2}{M^4} \sim \frac{m_{soft}^2}{M^2} \quad (2.2)$$

Note that the quadratic correction due to this term to the lightest Higgs boson mass is $\mathcal{O}(m_{soft}^2)$, and thus the mass shift is not inordinately high even if one has large M . A similar observation can be made even if one has

$$\lambda_{hard} \sim \frac{F}{M^2}, \quad (2.3)$$

as the net loop-induced contribution in a gauge-invariant framework finally turns out to be $\mathcal{O}(m_{soft}^2)$. This can be understood from the fact that the SUSY-breaking operators with such couplings give only holomorphic corrections to the MSSM Higgs potential [2]. However, if one turns completely model independent, and assumes that a quartic SUSY breaking term proportional to a purely phenomenological parameter λ is induced in the effective theory below the scale M , then the shift in the lightest Higgs mass proportional to M^2 cannot in general be avoided. Thus, the reconciliation between phenomenological hard SUSY breaking terms and a manageable shift in the Higgs mass is best achieved if the scale relevant for SUSY breaking is within an order above a TeV. At the same time, the μ parameter, which needs to be at best about a TeV, no more raises a naturalness issue.

There are two relevant scales determining the SUSY-breaking parameters, namely, \sqrt{F} and M . The only phenomenological input that we have is that $m_{soft} = \mathcal{O}(1 \text{ TeV})$, which is required if TeV-scale SUSY is to be a solution to the naturalness problem. As has been observed before in models of low-scale SUSY breaking, the MSSM assumption of a hierarchy of scales is not really necessary and therefore the hard terms can also be non-negligible. Thus for example, even at the tree level, the Higgs quartic coupling can get enhanced and the theoretical upper bound on the lightest Higgs mass modified accordingly.

Here we consider an extra-dimensional scenario in which both the scales \sqrt{F} and M are of similar order, on the order of several TeV's. In studies of this kind, the 'bulk' Planck mass is hypothesised as the source of the scale M . As has been already mentioned, we seek an alternative scenario where gravity does not have the primary role; instead, it is the modular fields which guide us to the scale of the effective theory related to the radius of compactification.

In the framework considered here, the universe is $(4+n)$ dimensional with $n \geq 1$ extra compact spacelike dimensions. While gravity can propagate in the new dimensions, the standard model (SM) fields are assumed to be localized on a 3-brane in the higher-dimensional space. The radii of these compact dimensions act like modular fields (T_i) and the stable radii of these extra dimensions are related to vev's $\langle T_i \rangle$ of these modular fields. As the compact dimensions are considered to be large, generically $\langle T_i \rangle \ll M_{Pl}$, where M_{Pl} is the four-dimensional Planck scale. Now, SUSY is assumed to be broken by the vev's of the F-term components of the fields T_i , $\langle F_{T_i} \rangle$. The scale of the effective 4-d theory in the visible sector however is set by $\langle T_i \rangle$. With both $\langle T_i \rangle$ and $\langle F_{T_i} \rangle$ an order or two above the TeV scale, the phenomenology of the 'effective' MSSM is controlled by a low-lying scale, with all the merits that have been mentioned above.

In order to have $m_{soft} = \mathcal{O}(1 \text{ TeV})$, where the mediation scale $M \sim 10 \text{ TeV}$, we require $F \sim 10 \text{ TeV}^2$, i.e., $\sqrt{F} \sim 3 \text{ TeV}$. This is a constraint that our model has to satisfy. For simplicity we assume a common stable radius for all the extra dimensions and therefore a common vev for all the moduli fields. Therefore, $\sqrt{\langle F_T \rangle}$, has to be of the same order as $\langle T \rangle$. In section III we shall construct an illustrative model and show that such a thing is easily achievable.

It should also be mentioned that such a scenario has a testable difference from one where the low-valued bulk Planck mass (such as in the scenario proposed by Antoniadis, Arkani-Hamed, Dimopoulos and Dvali [11]) is the scale of the effective theory. In the latter situation, the convolution of any amplitude involving the emission of gravitons by the density of graviton states results in the total amplitude for graviton emission suppressed by the bulk Planck scale only, thus raising hopes for the signatures of gravitons at the LHC. In the situation we consider, the bulk Planck mass is considerably higher. As a result, missing energy signals involving gravitons are not going to be visible, although all SUSY signals appropriate for the mass spectrum are predicted as usual.

Before we go into the Higgs spectrum, we outline some features of our proposal in the next section. Some related approaches, also based on SUSY breaking via extra dimensions, can be found in [12]. There have also been studies on the modification of lightest Higgs boson mass in the context of SUSY theories formulated in higher space dimensions [13].

III. A SPECIFIC MODEL

We now illustrate our analysis with a model which can be motivated in an underlying string-inspired supergravity theory [14].

The model has a KKLT-like [10] setup where light modulus, namely the volume modulus T , is stabilized by non-perturbative effects like gaugino condensation [15] leading to an $N = 1$ supersymmetric anti-de-Sitter (AdS) vacuum. This AdS vacuum is then uplifted to a SUSY breaking Minkowski (or de-Sitter) vacuum by branes which break $N = 1$ SUSY explicitly [16]. The vev of the modulus T as well as the corresponding F -term can be set to desired values, which in turn can generate soft terms in the visible sector at the TeV scale.

In our model, the effective $N = 1$ SUGRA contains the T -modulus, and the effective description breaks down at the compactification scale M_c , which, for a large radius (or light moduli) compactification may be set to be about 10 TeV.

Following KKLT proposal we introduce a SUSY breaking $\overline{D3}$ brane which uplifts the AdS vacuum to a dS vacuum where the vev of the T field (related to M_c) is primarily determined by the $N = 1$ SUSY sector. In our model, the other modulus field like the dilaton is assumed to have a large vev and hence it decouples from the theory below the scale $\langle T \rangle$.

The T -sector has the Kahler potential K and the superpotential W given by

$$K = -3 \ln(T + \overline{T}) \quad (3.1)$$

and

$$W = W_0 - Ae^{-aT}, \quad (3.2)$$

where $T = t + i\tau$, and T , the Kahler potential and the superpotential have been appropriately scaled to make them dimensionless. Here τ is the axion present in the theory. Since the Kahler potential depends only on t , therefore, the overall phase of W is irrelevant. Moreover, the relative phase between W_0 and A can be eliminated by shifting the axion field τ such that

$$\langle \tau \rangle = 0. \quad (3.3)$$

The condition for unbroken SUSY is given by,

$$\langle D_T W \rangle = 0, \quad (3.4)$$

where

$$D_T W = \partial_T W + \frac{\partial K}{\partial T} W. \quad (3.5)$$

Solving for W_0 , we get

$$W_0 = \langle Ae^{-at} (1 + \frac{2at}{3}) \rangle. \quad (3.6)$$

Using eqn. 3.6, we can write the following approximate relation determining $\langle t \rangle$ (which is equal to $\langle T \rangle$, as the axion vev is zero in this case) in terms of the parameters a , A and W_0 :

$$\langle at \rangle \simeq \ln \frac{A}{W_0}. \quad (3.7)$$

The scalar potential for the T field can be calculated using the following expression

$$V = M_{Pl}^4 e^K [K^{T\overline{T}} |D_T W|^2 - 3|W|^2], \quad (3.8)$$

where $K^{T\overline{T}} = (\frac{\partial^2 K}{\partial T \partial \overline{T}})^{-1}$. It can be shown that, at the SUSY preserving vacuum, defined by eqn. 3.4, the minimum of the potential takes the value

$$\langle V \rangle = -3m_{3/2}^2 M_{Pl}^2, \quad (3.9)$$

which is an AdS vacuum. Here, $m_{3/2}$ is the gravitino mass.

To stabilize T , we now introduce a $\overline{D3}$ brane which gives rise to a lifting term D/t^n in the scalar potential. For small values of t , this gives a large contribution to the potential.

SUSY is now broken in this hidden sector with an F -term vev given by

$$F_T \simeq \frac{n}{a} m_{3/2}. \quad (3.10)$$

In order to obtain soft SUSY breaking masses of the order of 1 TeV, we need to impose the condition that $m_{soft} \sim F_T/\langle T \rangle \sim 1$ TeV. If, in addition, we take $m_{3/2} \sim 1$ TeV, we see from eqns. 3.7 and 3.10 that for $n = 2$, we shall need W_0 and A to satisfy the relation

$$\ln \frac{A}{W_0} \sim 2. \quad (3.11)$$

Such values of W_0 and A are easily achievable in the framework we consider. Also note that, by making a suitable choice of the parameter a , one can also obtain the desired values for F_T and $\langle T \rangle$. For example, with $a = 0.2$ TeV^{-1} , we can get $\langle T \rangle = 10$ TeV, and correspondingly, $F_T = 10$ TeV^2 , as required.

Thus, we find by constructing this illustrative model that one can obtain a suitable low-scale SUSY breaking framework where the vev of the modular fields can act as the effective scale of suppression for the soft and hard SUSY breaking operators. Similar conclusion can also be obtained in other scenarios where the dilaton sector also acquires a superpotential from gaugino condensation.

IV. UPPER BOUND ON THE LIGHTEST NEUTRAL HIGGS MASS

We have outlined above a scenario where the vev of the T fields sets the scale of suppression for nonrenormalisable terms responsible for soft as well as hard SUSY beaking in the visible sector. Before we go on to examine the modified upper bounds on the lightest neutral Higgs mass in such a scenario, let us recall the salient features of the MSSM Higgs sector [17]. One requires two $SU(2)_L$ Higgs doublets, H_2 and H_1 , with hypercharge ± 1 . The scalar potential receives contributions from the F-terms, D-terms and the soft SUSY breaking terms. The part of the tree-level potential exclusive to the electrically neutral components of the Higgs fields $H_{1,2}^0$ is given by

$$V_{MSSM}^{H^0} = \overline{m}_1^2 |H_1^0|^2 + \overline{m}_2^2 |H_2^0|^2 - \overline{m}_3^2 (H_1^0 H_2^0 + h.c.) + \frac{1}{8} (g_1^2 + g_2^2) (|H_1^0|^2 - |H_2^0|^2)^2 \quad (4.1)$$

with $\overline{m}_{1,2}^2 = |\mu|^2 + m_{H_{1,2}}^2$ and $\overline{m}_3^2 = B\mu$ where $m_{H_{1,2}}^2$ and B are soft SUSY breaking parameters and μ is the Higgsino mass term in the superpotential. Also, g_1 and g_2 are the $U(1)_Y$ and $SU(2)_L$ gauge couplings respectively.

It has been mentioned that, in addition to the soft SUSY breaking contributions included in the above potential, one can in principle have additional hard SUSY breaking terms as well. This results in additional quartic terms in the superpotential, whose coefficients are not related to the gauge couplings. Such dimensionless SUSY breaking couplings do not, however, give rise to inordinately high quadratic corrections to the Higgs mass, so long as they are proportional to powers of F/M , although they can have more catastrophic consequences in the most general situation. These couplings can become particularly important in models of low-scale SUSY breaking where they are not suppressed by a very high-scale.

All possible renormalizable supersymmetry breaking interactions have been classified, for example, in [6]. We concentrate on the hard SUSY breaking terms arising only in the Higgs sector of the Lagrangian. This is partly for the sake of simplification, and partly due to the fact that the modified bounds on the Higgs mass suggested by us depend on other hard SUSY breaking terms only at higher orders of perturbation.

Following ref. [6], we consider the possible hard SUSY breaking interactions involving the Higgs scalar fields. In the SUSY Higgs sector, the possible gauge-invariant terms are given by

$$- \mathcal{L}_{Hard} = \frac{F}{M^2} [(H_2 \cdot H_1)^2 + h.c.] + \frac{|F|^2}{M^4} [(H_1^\dagger H_1)^2 + (H_2^\dagger H_2)^2 + (H_1^\dagger H_1)(H_2^\dagger H_2) + (H_1^\dagger H_2)(H_2^\dagger H_1)] \quad (4.2)$$

In the term proportional to $\frac{|F|^2}{M^4}$, we see that all possible quartic terms involving the Higgs fields in the MSSM scalar potential can now arise via the dimensionless SUSY breaking couplings also. Note that, in the MSSM scalar potential, these terms arise from the D-term contributions of the $SU(2)_L$ and $U(1)_Y$ interactions. Thus the coefficients of the quartic terms are determined entirely in terms of the gauge couplings g_1 and g_2 (see eqn. 4.1). In contrast, the above

SUSY breaking quartic terms reintroduce one-loop contributions to the Higgs mass(es), which can potentially be quadratically divergent.

Due to the addition of these new hard SUSY breaking terms given in eqn. 4.2, the tree-level MSSM scalar potential involving the neutral components of the Higgs doublets gets modified. The modified potential, V^{H^0} , can now be written as

$$V^{H^0} = V_{MSSM}^{H^0} + V_{Hard}^{H^0}, \quad (4.3)$$

where

$$V_{Hard}^{H^0} = \epsilon_1[(H_1^0 H_2^0)^2 + h.c.] + \epsilon_2(|H_1^0|^4 + |H_2^0|^4 + |H_1^0|^2 |H_2^0|^2). \quad (4.4)$$

Here we have defined

$$\begin{aligned} \epsilon_1 &= \frac{F}{M^2} \\ \epsilon_2 &= \frac{|F|^2}{M^4}. \end{aligned} \quad (4.5)$$

We now assume that at the minimum of the potential V^{H^0} the neutral components of the two Higgs fields develop vacuum expectation values

$$\langle H_1^0 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}}. \quad (4.6)$$

In order to trigger electroweak symmetry breaking at the right scale, v_1 and v_2 must satisfy the relation

$$(v_1^2 + v_2^2) = v^2 = \frac{4M_Z^2}{g_1^2 + g_2^2} = (246 \text{ GeV})^2. \quad (4.7)$$

Given the parameters in the soft and hard SUSY breaking sectors, v_1 and v_2 can be determined by using the potential minimization conditions:

$$\frac{\partial V^{H^0}}{\partial H_1^0} = \frac{\partial V^{H^0}}{\partial H_2^0} = 0, \quad (4.8)$$

which in this case translate to:

$$\overline{m}_1^2 - \overline{m}_3^2 \tan \beta + \frac{M_Z^2}{2} \cos 2\beta + \epsilon v^2 = 0, \quad (4.9)$$

$$\overline{m}_2^2 - \overline{m}_3^2 \cot \beta - \frac{M_Z^2}{2} \cos 2\beta + \epsilon' v^2 = 0, \quad (4.10)$$

where

$$\tan \beta = \frac{v_2}{v_1}, \quad (4.11)$$

$$\epsilon = (\epsilon_1 + \frac{\epsilon_2}{2}) \sin^2 \beta + \epsilon_2 \cos^2 \beta \quad (4.12)$$

$$\epsilon' = (\epsilon_1 + \frac{\epsilon_2}{2}) \cos^2 \beta + \epsilon_2 \sin^2 \beta \quad (4.13)$$

All the above relations have been written with the assumption that there is no CP-violation in the Higgs sector at the tree level. Therefore, all the relevant parameters and in particular, the vev's of the Higgs fields can be chosen as real.

The mass matrix for the Higgs bosons \mathcal{M}_{ij}^2 can be computed using the following relation

$$\mathcal{M}_{ij}^2 = \frac{\partial^2 V^{H^0}}{\partial H_i^0 \partial H_j^0} \Big|_{\langle H_1^0 \rangle = \frac{v_1}{\sqrt{2}}, \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}}} . \quad (4.14)$$

Following the usual convention in MSSM, we express the Higgs mass eigenvalues in terms of two free parameters in the Higgs sector, which we take to be M_A and $\tan \beta$. We then consider the bound on the Higgs mass in the so-called decoupling limit where $M_A \gg M_Z$, and the expression for the lightest neutral Higgs mass has a rather simple form. Later in this section, where we present numerical results in Table I, we show the Higgs mass shifts, based on an exact numerical calculation, away from the decoupling limit. We now need to calculate the pseudoscalar Higgs mass M_A . The neutral pseudoscalar mass matrix is given by

$$\mathcal{M}_{Im}^2 = \begin{pmatrix} \overline{m}_3^2 \tan \beta - 2\epsilon_1 v^2 \sin^2 \beta & \overline{m}_3^2 - 2\epsilon_1 v^2 \sin \beta \cos \beta \\ \overline{m}_3^2 - 2\epsilon_1 v^2 \sin \beta \cos \beta & \overline{m}_3^2 \cot \beta - 2\epsilon_1 v^2 \cos^2 \beta \end{pmatrix} \quad (4.15)$$

This mass matrix has one null eigenvalue, corresponding to the Goldstone boson which ultimately lends a longitudinal component to the Z boson. The other eigenstate is the pseudoscalar Higgs whose mass is given by

$$M_A^2 = \overline{m}_3^2 (\tan \beta + \cot \beta) - 2\epsilon_1 v^2. \quad (4.16)$$

The mass-matrix for the CP-even neutral Higgs bosons is now given by:

$$\mathcal{M}_{Re}^2 = \begin{pmatrix} (M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta) + 2v^2(\epsilon_1 \sin^2 \beta + \epsilon_2 \cos^2 \beta) & -(M_A^2 + M_Z^2) \sin \beta \cos \beta + \epsilon_2 v^2 \sin \beta \cos \beta \\ -(M_A^2 + M_Z^2) \sin \beta \cos \beta + \epsilon_2 v^2 \sin \beta \cos \beta & (M_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta) + 2v^2(\epsilon_1 \cos^2 \beta + \epsilon_2 \sin^2 \beta) \end{pmatrix} \quad (4.17)$$

Diagonalization of this mass-matrix will give the eigenvalues for the neutral Higgs bosons:

$$m_{H_0, h_0}^2 = \frac{1}{2} [Tr \mathcal{M}_{Re}^2 \pm \sqrt{(Tr \mathcal{M}_{Re}^2)^2 - 4 Det \mathcal{M}_{Re}^2}]. \quad (4.18)$$

We can calculate the lightest CP-even Higgs mass in the decoupling limit from eqns. 4.17 and 4.18. We find that

$$m_{h_0}^2 = M_Z^2 \cos^2 2\beta + 2v^2 [2\epsilon_1 \cos^2 \beta \sin^2 \beta + \epsilon_2 (\cos^4 \beta + \cos^2 \beta \sin^2 \beta + \sin^4 \beta)] \quad (4.19)$$

Thus at the tree level itself, the Higgs mass gets enhanced due to the additional hard SUSY breaking terms. To this we must add the loop corrections. In the absence of the dimensionless SUSY breaking terms, the dominant correction to the lightest Higgs boson mass comes from the contribution of the top quark and top squark loops. With the assumption of a small mixing among the gauge eigenstates in the top squark sector and with masses $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ much greater than the top quark mass m_t , one finds a large positive one-loop radiative correction to the Higgs mass [18]:

$$\Delta(m_{h_0}^2) \simeq \frac{3}{2\pi^2} \frac{m_t^4}{v^2} \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right). \quad (4.20)$$

Due to the presence of dimensionless hard couplings in the theory, in addition to this there will be corrections quadratically sensitive to the cut-off scale. They are generically given by:

$$\delta m_{h_0}^2 \sim \frac{\lambda_{hard}}{16\pi^2} \Lambda^2, \quad (4.21)$$

where λ_{hard} is a generic dimensionless hard coupling giving rise to quadratic one-loop corrections, and Λ is the cut-off of the theory.

In the case that we consider, we see from eqn. 4.4 that the correction can be estimated to be

$$\begin{aligned} \delta m_{h_0}^2 &\sim \frac{\epsilon_2}{16\pi^2} M^2 \\ &= \frac{1}{16\pi^2} \frac{|F|^2}{M^4} M^2 \\ &= \frac{m_{soft}^2}{16\pi^2}. \end{aligned} \quad (4.22)$$

Note that, although this correction comes from a quadratically divergent one-loop amplitude, it is bounded from above by the soft SUSY-breaking mass squared, and is also independent of the cut-off scale as long as the dimensionless couplings are of the given form (in particular determined in terms of F and M and not arbitrary).

Adding up all the contributions from eqns. 4.19, 4.20 and 4.22 in quadrature we find that the Higgs mass for $\tan\beta \rightarrow 0$ can be as large as 155 GeV for a soft SUSY breaking scale $m_{soft} \sim 1$ TeV and $M = 10$ TeV. Thus the low SUSY breaking scale engineered by the extra compact dimension(s) leads to a substantial enhancement of the Higgs mass upper limit, so much so that the lightest neutral scalar in SUSY may even be detected via the ‘gold-plated’ ZZ channel. It is also to be noted that the dependence of the Higgs mass shift on $\tan\beta$ is rather weak, unless the cut-off scale M is as low as 1 TeV [9].

In order to see the dependence of the lightest Higgs mass on the cut-off scale M , for a fixed value of M_A , $\tan\beta$, and m_{soft} , we diagonalize the CP-even scalar mass matrix in eqn. 4.17 numerically for different values of M and present the results in Table I. F_T has also been varied accordingly along with M in order to keep m_{soft} fixed at 1 TeV. From Table I we can see that for lower values of M , in the range of 1 – 5 TeV, the enhancement in the lightest Higgs mass is rather dramatic. Even the tree-level Higgs mass gets corrected by a large amount. This is because the hard SUSY-breaking quartic couplings in these cases become considerably large (for e.g., $\epsilon_1 \sim \mathcal{O}(1)$ for $M \sim 1$ TeV) and comparable to, or more than, the MSSM quartic coupling. In addition to the well known 1-loop correction due the top-stop loops, there is also now an additional 1-loop contribution coming from the quadratically divergent graphs, giving a further upward shift to the Higgs mass. This correction, as noted before, is independent of the cut-off scale M , which is an important feature of the form of the dimensionless couplings involved. Thus, even if the cut-off scale is as high as 50 TeV, we obtain at least ~ 20 GeV shift to the Higgs mass compared to the 1-loop prediction in MSSM. In addition, we can see from eqns. 4.16 and 4.19 that, due to the dimensionless SUSY breaking couplings, the scalar and pseudoscalar masses at the tree level receive corrections of opposite sign. Thus, in our scenario while the Higgs mass is predicted to increase compared to the MSSM value, the pseudoscalar mass is expected to become lower.

M (TeV)	$m_{h_0}^{tree}$ (GeV)	$m_{h_0}^{tree+1-loop}$ (GeV)
1	361	382
5	115	170
10	95	156
50	83	150

TABLE I: Change in mass of the lightest Higgs boson with variation in the cut-off scale M . The masses have been calculated with $M_A = 200$ GeV, $\tan\beta = 5$ and $m_{soft} = 1$ TeV. F_T has been varied concomitantly with M .

Finally, let us also note that the model considered by us here does not involve any additional matter or gauge fields. In that sense, we still work in a “minimal” framework, where the SUSY-breaking terms are suppressed by a lower scale and therefore the effect of hard SUSY-breaking operators become important. And we observe that it is possible to obtain a substantial enhancement in the lightest Higgs mass, thereby relaxing the stringent upper bound found in MSSM. In addition, if the cut-off scale is rather low, even the tree-level Higgs mass gets significantly shifted, thus alleviating the tension between the LEP limit and the MSSM bound on the Higgs mass [3]. In other approaches to solving this so-called fine-tuning problem within the MSSM, there have been studies where the MSSM is extended, for example, by an additional $U(1)$ gauge group, accompanied by a singlet scalar field coupling to the two Higgs doublets by a superpotential term [5]. Although the upper bound on the tree-level lightest CP-even Higgs doublet mass is raised in these models, too, the actual mass eigenvalues are generally smaller because of mixing with the singlets. The tension with the LEP-bound is still avoided as the LEP limits themselves change due to modifications in the relevant couplings [5]. Therefore, we note that the Higgs mass eigenvalues predicted in these extensions of the MSSM often have much lower magnitude than those obtained in our scenario. This can be used as a discriminating feature of our scenario with these gauge and singlet extensions of the MSSM. Besides, the scheme proposed by us is offered as an explanation of a situation where the signals of SUSY (say, in the form of large missing transverse momentum and jets/leptons) are found at the LHC, but the lightest neutral Higgs mass reconstructed turns out to be considerably higher than what is normally expected within the MSSM.

V. SUMMARY AND CONCLUSION

We have considered a scenario where the soft SUSY-breaking terms are generated through higher-dimensional operators suppressed by energy scales a little above a TeV. At the same time, possible hard SUSY breaking terms have been retained, which can contribute to the Higgs mass(es) at tree-level as well as through loop effects. This is a generic situation that can be expected if new physics is always staring one in the face as one goes continuously upwards in energy, rather than there being some ‘desert’ above the TeV order. We demonstrate the viability of our

proposal in an illustrative model based on extra compact spacelike dimensions. Here one indeed obtains such a scale, corresponding to the stable vev of the scalar component of the modular fields connected with the extra dimensions. A scalar potential has been explicitly constructed where both the SUSY breaking F-term vev (F_T) and the modular field vev ($\langle T \rangle$) lie in the range of a few TeV's, and give rise to TeV-scale soft breaking parameters. The vev of T sets the scale of suppression of higher-dimensional operators at a scale below $\langle T \rangle$ when T is integrated out, and thus generates the predominant SUSY-breaking effects.

We show that, in such a setting, the lightest even-parity neutral Higgs mass can receive two types of additional contributions: one at the tree-level due to the hard SUSY-breaking term(s), and the other at the loop level, dictated by the new cut-off scale of 10 TeV or so. Our numerical estimate shows that, over the usual, phenomenologically allowed region of the MSSM parameter space, this leads to considerable upward revision of the upper limit of the lightest Higgs mass. While for $\langle T \rangle \simeq 1$ TeV, the corrected mass limit can be as large as about 380 GeV, more modest, but significant, revisions upto 150 – 170 GeV are quite possible for cut-off scales upto 10 TeV. Thus, if signals of SUSY, perhaps in the form of large missing energy and jets/leptons, reveal themselves at the Large Hadron Collider (LHC), and at the same time one fails to find the Higgs boson with mass below 130 GeV or so, the situation is, after all, not irreconcilable. One should then look seriously at the possibility of the clue to SUSY breaking lying a little above the reach of the LHC, with possible indirect manifestations in other experimental results. The fact that the fields connected with the fluctuating radii of extra dimensions can be responsible for such large shift in the Higgs mass injects an added degree of richness to such a scenario.

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